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APEX: Positive evidence for sharp 800 keV pairs from heavy ion collisions near the Coulomb barrier

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Abstract

The best fit to the published APEX U+Th data describes a sharp line of 123 pairs of width 23 keV at 793 ± 7 keV; also, the data impose a positive 99.0%CL *lower* bound of 23 sharp pairs. It is therefore untenable to argue from the APEX data against the existence of sharp pairs. Data-only ratios APEX/EPOS pair counts show empirically that the two experiments' pair data are mutually consistent and of comparable statistical potency: conflicts, if any, can be resolved only by other independent evidence. A perspective is offered on the current status of the Sharp Lepton Problem, and alternative non-heavy-ion experimental approaches to it are recommended.

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I. INTRODUCTION

The APEX experiment was mounted to test EPOS' earlier observations of sharp (e^+e^-) pairs from high-Z heavy ion collisions. In a recent letter the APEX collaboration [1] presents its results, and considers the earlier EPOS U+Th results [2–5] in the light of their own new data. It asserts that their data offers no statistically significant evidence of sharp pairs and concludes “that the results of the present experiment represent a real disagreement with the previous observations”.

Since the present report draws a seemingly contrary conclusion from the very same data, it is essential to stipulate at the outset that the APEX (U+Th) data does unquestionably exclude an excess sharp pair count near 800 keV as large as the ≈ 2560 counts which they were expecting. In fact the APEX report establishes a specific (99%CL) upper bound¹ of 292 excess counts near 800 keV. That discrepancy leads the APEX collaboration to question the existence of the previously reported sharp pairs, and in particular of the EPOS line near 800 keV, and to draw the conclusion quoted above, of a “real disagreement” with the EPOS results.

The present analysis² confirms these results, and prescribes a range of even smaller upper bounds upon the excess pair counts near 800 keV: from 217 to 252 counts. Nevertheless, the present analysis also provides statistically significant positive evidence that APEX actually counted 123 ± 46 sharp pairs near 800 keV. This inference contradicts none of APEX empirical evidence. In particular it honors the quantitative upper bound of the APEX report. In the end, we find that the APEX U+Th experiment recorded (at 793 ± 7 keV) 123 ± 46 sharp pairs, above a background of 1480 pairs/20keV, among a total of 40.8K EPOS-type³ pairs. The EPOS experiment recorded (at 809 ± 8 keV) 97 ± 38 sharp pairs⁴ above a background of 1280 pairs/20keV among a total of 50K pairs. Thus under direct comparison, the APEX and the EPOS pair data sets are of comparable size, and yield comparable numbers of background pairs and of sharp pairs above background near 800 keV. There is therefore no significant experimental evidence of contradiction between them. It follows also that

¹APEX presents its upper bounds as cross sections. We prefer to present the discussion in terms of counts, in order to keep it independent of assumptions about the energy dependence of the sharp pair production cross section. We use the conversion factor, $2560/5=512$ excess counts/($\mu b/sr$), to convert from APEX' sharp pair production cross section to sharp pair counts. See also footnotes No. 7 and No. 26.

²Brief summaries of the present work have been presented in Refs. [6,7].

³“EPOS'-type” pairs are those which are Qualitatively Similar to the pairs accepted by the EPOS apparatus, as discussed in more detail in Section V.A below.

⁴This is the count given in Ref. [8] of excess pairs above the full (five run) EPOS background without the Time of Flight selection. It may be compared with the 105 ± 20 counts of Ref [5] extracted from the two highest energy runs alone. Without the EPOS Time-of-Flight gating, the latter count increases to 113 ± 28 , according page 156 of Ref. [5].

as regards coincident pairs neither experiment can claim clear statistical superiority for resolving any differences in their pair distributions.

We note that we consider these pairs to be of very special interest because they suggest [9–14] the unexpected existence of a tightly bound Quadronium ($e^+e^+e^-e^-$) particle, Q_o , which might affect [13] the discrepant 3γ decay of positronium, and the scattering [14] of few MeV electrons, positrons, and of ~ 1.8 MeV photons, from high-Z nuclei, as discussed further in Section VI.

II. APEX' PUBLISHED PAIR SUM ENERGY DATA

Figure 1 summarizes the published⁵ APEX data on 22K (e^+e^-) pairs⁶ from its U+Th study with sum energies in the range from 200 to 1400 keV. It presents the number of counts observed in each 20 keV bin, and compares it with APEX' background distribution of pairs which results from random electron-positron coincidences. The shape of the latter “event-mixed” distribution was measured by APEX by taking a random positron from one event together with an electron from the subsequent event⁷. Also shown is the APEX' simulated sharp pair distribution, N_{SSP} , tabulated in Table I, reduced by a factor of 10 from the $5\mu\text{b}/\text{sr}$ value expected by APEX. All of the data of Figure 1 were presented graphically in Figure 2a of the APEX report, and in Figs. VI.1.2, and VI.1.3 of the thesis of M.R. Wolanski [15] on the APEX U+Th experiment.

⁵The author is grateful to the APEX collaboration for supplying the data published in Figure 2 of Ref [1] in numerical form.

⁶These are pairs selected by APEX to resemble those accepted by the EPOS experiment: The positron and the electron must be observed in opposite arms of the detector, (“RL” pairs), and the lepton energies must fall into a “wedge cut” similar to (but not identical with) that imposed by EPOS to enhance its sharp pair signal. We note however that these APEX pairs are not “Qualitatively Similar” EPOS-type pairs (in the sense discussed in Section V.A), because they include “(1,n)” pairs from events in which any number, n , of electrons may be detected in coincidence with the positron, whereas EPOS accepted only “(1,1)” pairs from events in which one and only one electron is coincident with the positron.

⁷The true smooth pair background also includes pairs from nuclear IPC processes [4,5]. Moreover, if Quadronium decay occurs [13,14], background pairs (as well as energetic photons easily mistaken for nuclear gamma rays) may also arise from its most probable decay to (e^+, e^-, γ) . For these and other reasons, the present event-mixed background is subject to an unknown systematic error not included in the analysis. Nevertheless, for simplicity, and to register our results most directly with those of the APEX report, we here take the APEX event-mixed distribution of Table I, without adjustment, to describe the smooth pair background in the region of interest.

A. Sharp Pairs Near 800 keV

The present analysis focusses upon pairs in the energy range from 500 to 900 keV, where earlier studies [2,5,16–18] have indicated the occurrence of pairs of sharp sum energy. Table I presents the relevant APEX U+Th data in numerical form. It lists twenty-one sum energy bins from 510 to 910 keV, and their observed pair counts, N_{OBS} . Columns three through seven present the APEX' event-mixed background distribution, N_{BK} , its simulated⁸ sharp pair distribution, N_{SSP} , the number of “Excess” (above the background) sharp pairs, N_{EXC} , the values of $\chi_i^{APX} = (N_{OBS} - N_{BK})/(N_{OBS})^{1/2}$, and the APEX inferred upper bound upon the pair production cross section. Columns eight and nine list the values of our best fitting “One Sharp Line” distribution, N_{FIT} , and of its deviations, χ_i , from N_{EXC} in units of the standard deviation, $\sigma = (N_{OBS})^{1/2}$, and columns ten and eleven list the 99% CL lower and upper bounds upon the mean sharp pair count in each bin, calculated as described below.

Table I also provides the chi-squared sum for testing this APEX data against the hypothesis that it consists only of APEX' event-mixed background. For these twenty one bins⁹ the value of chi-squared is 28.30, so that the reduced chi-squared value for 20 degrees of freedom has an acceptable value of 1.41. Although this Background-Only assumption provides a moderately good fit to this data, we shall see that the assumption of one sharp line yields a substantially better fit, tabulated in columns eight and nine.

Figure 2 plots in the 500-900 keV range of Figure 1 the observed pair count, N_{OBS} , measured by APEX (from column 2 of Table I). It also exhibits as “error bars”, the standard deviations, $\pm\sigma = (N_{OBS})^{1/2}$, which measure the fluctuations expected in each bin count due to Poisson statistics alone. Also plotted in Figure 2 is the APEX' event-mixed background (solid line) and the added (cross hatched) contribution of the best fitting, “One-Sharp-Line” distribution, from columns three and eight of Table I, respectively. (The fitting process is detailed below.)

⁸This is the distribution plotted in Figure 1 of the 2914 counts computed by APEX' simulation program to occur if a 1.822 MeV source of sharp pairs were produced with an average cross section of $5\mu\text{b}/\text{sr}$, and decayed at rest in the C.of M. frame. Our gaussian best fit to it has 2560 counts, a width of 18.9 keV, and a maximum at 804.3 keV. See also footnote No.1.

⁹For the sixty 20 keV bins (with energies 210 keV to 1390 keV) presented in Figure 1, the corresponding χ^2_{59} value is 65.76, yielding a reduced chi-squared value of 1.11. The APEX' background therefore serves better to describe APEX pair data over the full 210-1290 keV APEX energy range than over the 510-910 keV range considered here.

TABLE I: U+Th PAIR DATA, FITS and BOUNDS

APEX' RESULTS: Measured, Simulated and Inferred							One Line Best Fit		99%CL Bounds	
E_{SUM}	N_{OBS}	N_{BK}	N_{SSP}	N_{EXC}	$(\chi_i)^{APX}$	$(d\sigma/d\Omega)^{UB}$	N_{FIT}	χ_i	ν_L	ν_U
510	610	608.7	5.3	+1.3	+0.1	0.80	0.0	+0.1	0.3	66.7
530	634	629.0	4.0	+5.0	+0.2	0.72	0.0	+0.2	0.4	70.6
550	653	641.2	2.6	+11.8	+0.5	0.68	0.0	+0.5	0.5	76.6
570	635	647.3	2.6	-12.3	-0.5	0.45	0.0	-0.5	0.2	59.0
590	640	646.7	1.3	-6.7	-0.3	0.44	0.0	-0.3	0.3	62.8
610	607	643.1	4.0	-36.1	-1.5	0.28	0.0	-1.5	0.1	45.4
630	627	645.8	5.3	-18.8	-0.7	0.32	0.0	-0.7	0.2	54.8
650	591	637.4	21.2	-46.4	-1.9	0.21	0.0	-1.9	0.1	40.7
670	657	630.1	30.5	-26.9	+1.0	0.47	0.0	+1.0	0.9	89.8
690	650	628.1	22.5	-21.9	+0.9	0.42	0.0	+0.9	0.7	85.0
710	608	613.8	50.3	-5.8	-0.2	0.28	0.0	-0.2	0.3	61.8
730	616	618.8	62.2	-2.8	-0.1	0.28	0.0	-0.1	0.3	64.2
750	621	607.7	55.6	+13.3	+0.5	0.32	0.1	+0.5	0.5	76.2
770	614	602.1	95.3	+11.9	+0.5	0.30	11.8	0.0	0.5	74.8
790	682	599.0	748.1	+83.0	+3.2	0.57	83.1	0.0	24.1	145.4
810	613	584.5	1742.4	+28.5	+1.2	0.33	28.2	0.0	1.0	89.1
830	567	574.8	62.2	-7.8	-0.3	0.21	0.3	-0.3	0.2	58.2
850	512	561.5	4.0	-49.5	-2.2	0.12	0.00	-2.2	0.1	35.9
870	583	546.2	0.	+36.8	+1.5	0.32	0.00	+1.5	1.7	95.2
890	517	529.8	0.	-12.8	-0.6	0.17	0.00	-0.6	0.2	52.4
910	502	504.9	0.	-2.9	-0.1	0.18	0.00	-0.1	0.3	57.9
$(\chi^2_{20})^{APX} = \sum(\chi_i^{APX})^2 = 28.30$							$\chi^2_{17} = \sum(\chi_i^2) = 16.65$			
$(\tilde{\chi}^2_{20})^{APX} = 1.41$							$\tilde{\chi}^2_{17} = 0.98$			

Table I. The table lists the APEX pair data in the range 500-920 keV, from Figure 2a of Ref. [1], as follows. (1)The bin energies, E_{SUM} , (2) APEX' pair counts, N_{OBS} , (3) APEX' event-mixed pair background, N_{BK} ,and (4) N_{SSP} , APEX' simulation calculation of the sharp pair distribution from back-to-back sharp pair decay at rest in the C. of M. frame of an object produced with a cross section of $5\mu\text{b}/\text{sr}$. Column five presents the excess pair count above the background, N_{EXC} , column six the values of χ_i^{APX} , whose squares sum to $(\chi^2_{20})^{APX}$, and column seven, APEX' upper bound (in $\mu\text{b}/\text{sr}$) on the pair cross cross section. Columns eight and nine provide our best Gaussian fit, N_{FIT} , to the excess in column 5, and the remnant deviations, χ_i , of that best fit from the observed values, whose squares sum to χ^2_{17} . The last two columns list ν_L and ν_U , the lower and upper bounds upon the mean excess counts in each bin implied (at the 99% confidence level) by the observed counts.

It is evident in Figure 2, that the largest excess pair count occurs in the bin centered at 790 keV, and that its magnitude (83.0 counts) is about 3.2σ . Such a large excess ought

statistically to occur only once in 500 such one-bin measurements¹⁰. Figure 2 requires that any attempt to argue on the basis of the APEX experiment against the occurrence of sharp pairs near 800 keV must address the significance of this 790 keV excess.

III. CHI-SQUARED ONE-SHARP-LINE ANALYSIS OF APEX' PAIR DATA

In fact, the assumption that a gaussian-shaped distribution of sharp pairs describes these excess counts allows a precise fit to this excursion, and effects a statistically significant reduction in the value of chi-squared. Therefore we compare with APEX' “Background-Only” hypothesis the alternative “One-Sharp-Line” hypothesis that the excess pair count is well explained by the sum, $N_{FIT}^{TOT} = N_{BK} + N_{FIT}$, of the event-mixed background and a distribution, N_{FIT} , arising from a single sharp pair line. Specifically, we define

$$N_{FIT}(E_i) = \int_{\Delta E_i} D(E; S, \Gamma, E_S) dE, \quad (1)$$

to specify the number of counts from the sharp line distribution which fall into the bin, ΔE_i , near E_i , and take the sharp pair energy distribution, D, to be of gaussian form centered at energy E_S , with FWHM of Γ , and with an integrated total number of pairs equal to S. These three parameters are chosen to minimize the chi-squared sum over all the bins, to which each bin contributes a term,

$$(\chi_i)^2 = (N_{OBS} - N_{TOT}^{FIT})^2 / (N_{OBS}), \quad (2)$$

where the values, χ_i , are listed in Table I (as are the corresponding values, χ_i^{APX} , for the APEX' Background-Only fit).

A. Chi-Squared Results for APEX' Sharp Pairs

The quantitative numerical results of the chi-squared analysis are summarized in Table II and Figure 3. One finds that the minimum value of chi-squared occurs at the values¹¹ ($S = 123.4$ counts, $\Gamma = 23.1$ keV, $E_S = 792.8$ keV), and has the value $\chi^2 = 16.65$, a substantial reduction from the Background-Only value of 28.30. Because of the three additional

¹⁰Since the APEX experiment consists of some 60 such single-bin measurements, one statistical fluctuation of this relative magnitude in an APEX experiment is a mildly improbable event.

¹¹We have also executed a four parameter (B, Γ, E_S, S) fit in which the coefficient, B, of the background term is allowed to vary, together with the three parameters of the sharp line. One finds the best fitting values ($B = 0.991, 25.7, 792.4, 140.6$) and $\chi^2 = 15.70$ ($\tilde{\chi}^2 = 0.981$), as compared with the values ($B \equiv 1.0, 23.1, 792.8, 123.4$) and $\chi^2 = 16.65$ ($\tilde{\chi}^2 = 0.979$) obtained here by keeping the background coefficient fixed at the APEX' value. Since the present fixed-background $B \equiv 1$ fit yields a smaller value for the total strength of the sharp pair line, it understates the case for the occurrence of such a line. It also simplifies the discussion by specifying one and the same background distribution for both fits.

parameters of the gaussian line, the χ^2 distribution now has only 17 degrees of freedom. The reduced chi-squared value, $\tilde{\chi}^2$, therefore decreases to 0.98 from the 1.41 value for the Background-Only fit. Clearly the One-Sharp-Line description is statistically superior to the Background-Only description presented in the APEX report. The statistical significance of that superiority is further quantified by means of Confidence Level analysis for upper and lower bounds, as discussed below. In addition, the $\pm 2.58\sigma$ χ^2 99% interval, into which repeated measurements should fall with 99% probability, extends from about 24 counts to 252 counts. The chi-squared analysis therefore implies it to be very unlikely that any repetition of the APEX experiment could fail to produce excess sharp pairs near 800 keV.

TABLE II: BEST χ^2 FIT to APEX' U+Th PAIR DATA				
	D_f =No. D.of F.	Fit Quality: χ^2 , $\tilde{\chi}^2 = \chi^2/D_f$;	Probability: $P_\chi(\chi^2/D_f)$	99% Sh.Pair Range: $N_{SP}^L \leq N_{SP} \leq N_{SP}^U$
APEX' Background-Only:	20	28.30	1.41	10.2% No Sharp Pairs
Background+Sharp Line:	17	16.65	0.98	47.8% $24 \leq 123 \leq 252$

Table II. The Table compares two fits to twenty one of the (e^+e^-) pair counts reported by the APEX collaboration in Ref. [1]. The three parameter Background+One-Sharp-Line fit is a better fit than APEX' one parameter Background-Only description, as discussed further in the text. These results support the hypothesis that an excess of sharp pairs occurs in the APEX data near 790 keV. Column six shows that one should expect a re-measurement of the 790 keV excess sharp pair count to yield a positive value in the range from 24 to 252 counts more than 99% of the time.

When the assumed description is in fact the true description, then the most probable reduced chi-squared value, $\tilde{\chi}^2$, is 1.0, and larger and smaller values are expected to occur in subsequent measurements with 50% probability. Table II shows that for the One-Sharp-Line distribution one expects a new measurement to yield a larger chi-squared value 47.8% of the time, as compared with the value of 10.2% for the Background-Only distribution.

IV. 99% CONFIDENCE LEVEL BOUNDS ON APEX' SHARP PAIRS

We next apply Confidence Level analysis to the APEX data of Table I and Figure 2, to set upper and lower limits upon the mean excess sharp pair counts. Specifically, we compute the bounds which the APEX data impose (at the 99% Confidence Level) upon the greatest mean value, ν_U , and upon the least mean value, ν_L , of excess sharp pairs for 20-, 40-, and 60- keV intervals, corresponding to the treatment of the APEX data bin by bin, and as sums of two and of three adjacent bins. The resulting bounds for the twenty one 20 keV bins are listed in columns ten and eleven of Table I.

The analysis assumes that the observed pair count in each bin is composed of the sum of a background contribution, n_B , arising randomly from a Poisson distribution of mean value, N_{BK} ,

$$P(N_{BK}; n_B) = (\exp - N_{BK})[(N_{BK})^{n_B}]/n_B!, \quad (3)$$

and a sharp pair contribution, n_{SP} , arising from a second Poisson distribution of mean, ν . The mean background value, N_{BK} , in each bin is taken from the APEX' best fitted event-mixed distribution (given in column three of Table I), and the value of ν is chosen to yield the pre-selected Confidence Level (CL), that a repeat of the APEX experiment would yield a count smaller than the observed count, N_{OBS} . As a function of the mean sharp pair count, ν , that probability is defined [19,20] as follows¹²:

$$\alpha(\nu; N_{OBS}) = \left[\sum_{n=0}^{N_{OBS}} (P(N_{BK} + \nu; n)) \right] / \left[\sum_{n=0}^{N_{OBS}} (P(N_{BK}; n)) \right] \quad (4)$$

As ν increases from zero to ∞ , the value of α decreases from 1 to zero. When ν is such that the quantity $\alpha(\nu; N_{OBS})$, has the value, 0.010, then that value is $\nu = \nu_U$, and is the 99.0% Confidence Level upper bound upon the true mean value, ν_{TRUE} : If ν_{TRUE} had the value ν_U , then 99% of repeated measurements would yield a smaller value for n than N_{OBS} . Likewise when $\alpha(\nu)$ assumes the value 0.990, the corresponding value of ν is equal to ν_L , the 99.0% CL *lower* bound upon the mean sharp pair count: If ν_{TRUE} had the value ν_L , then 99% of repeated measurements would result in a number, n , of counts greater than N_{OBS} .

Figure 3 plots the excess pair counts per 20 keV bin, N_{EXC} (from column five of Table I), defined as the difference, $N_{OBS} - N_{BK}$, of columns three and two, together with the standard deviations, $\pm\sigma_i = \pm N_{OBS}^{1/2}$, plotted as error bars. N_{EXC} exhibits a maximum¹³ of 83 ± 26.1 counts/20 keV for the bin at 790 keV, and is best described (in the χ^2 sense) by the gaussian one-line distribution, shown cross hatched. For this same 790 keV bin, the figure also exhibits (as triangles) the 99% CL bounds upon the mean sharp pair value implied by its 83 count excess. The upper bound of the 790 keV bin count is 145.4 counts/20keV.

However, the greater interest for the present discussion lies in the results for the *lower* bounds imposed by the data. For all of the energy bins except that at 790 keV, the lower limits listed in Table I have a negligible value (≤ 2 counts per 20 keV), indicating that the APEX data fails to provide “statistically significant” (i.e., at the 99% CL) evidence to support even a positive mean excess pair count even as small as two. But for the 20 keV bin centered at 790 KeV, the lower bound becomes substantial, with a value¹⁴ of 24.1 counts per 20 keV (about 0.9 σ).

In summary, Figure 3 shows that for the measured 3.2σ excess of 83 counts in the 20 keV bin at 790 keV, the Confidence Level analysis indicates a 99% CL lower bound of 24 counts upon the sharp pair mean, and a 99% CL upper bound of 146 counts. The APEX data therefore provides statistically significant evidence that a non-zero positive excess of sharp pairs above the background occurs in a 20 keV bin near 800 keV.

¹²The present form of α is given by Eq(II.28) of Ref. [19], which follows identically from Eqs.(2) and (7) of Ref. [20] by integration and summation.

¹³For the best fitting One-Sharp-Line distribution, this maximum in the one 20-keV bin count corresponds to a total sharp pair line strength of 123.3 counts, as given in Table III.

¹⁴For our best fit distribution, this corresponds to a lower bound of 35.4 counts upon the total line strength and an upper bound of 217.1, as given in Table III.

Analogous 99% upper and lower bounds have been established by considering also the counts/40keV and the counts/60 keV which arise by combining two and three adjacent 20 keV bins. The results are presented in Table III. For each bin grouping a total line strength also has been inferred by assuming our best fitting line shape. Each of the four analyses provides statistically significant evidence that the mean total excess sharp pair count near 800 keV is greater than 23 counts and less than 252 counts.

TABLE III: 99% CONFIDENCE LEVEL UPPER and LOWER BOUNDS on APEX' SHARP 800 keV PAIR COUNTS			
Data Grouping	Pair Energy(keV)	$\nu_L < (N_{EXC} \pm \sigma) < \nu_U$ (counts)	$\nu_L/\sigma < (N_{EXC}/\sigma) < \nu_U/\sigma$ (Stnd. Deviations)
Single Bin (20keV)	790	$24.1 < (83.0 \pm 26.1) < 145.4$	$0.9 < 3.2 < 5.6$
		$(f_1 = 0.673) \Rightarrow (\text{Total Line Strength} = 35.4 < 123.3 < 217.1);$	
Double Bin (40keV)	800	$29.4 < (111.5 \pm 36.0) < 197.8$	$0.8 < 3.1 < 5.5$
		$(f_2 = 0.901) \Rightarrow (\text{Total Line Strength} = 32.6 < 123.8 < 219.5);$	
Triple Bin (60 keV)	790	$23.4 < (123.4 \pm 43.7) < 226.3$	$0.5 < 2.8 < 5.2$
		$(f_2 = 0.997) \Rightarrow (\text{Total Line Strength} = 23.4 < 123.6 < 226.7);$	
χ^2 Best Fit*:	792.8	$24.0 < (123.4 \pm 45.8) < 251.1$	$0.5 < 2.7 < 5.5$
*(Here a $\pm \sigma_{\chi^2}$ shift increases χ^2 by +1, and the shift to the 99% Interval, by $+(2.58)^2$.			

Table III presents the (99% Confidence Level) lower and upper bounds, ν_L and ν_U , which the APEX data imposes upon the mean excess pair counts in the single 20 keV bin centered at 790 keV, in the two-bin combination centered at 800 keV, and in the three-bin combination centered at 790 keV. For all of the 20 keV bins other than that at 790 keV, the 99% CL lower bound is negligible (less than two counts), indicating that only near 790 keV does the APEX data *require* a positive pair excess. For each grouping, the table also presents a projected total pair count based upon the fraction f_n of excess pairs expected to occur in the interval in question if the excess pairs are distributed according to the gaussian One-Sharp-Line χ^2 best fit described in the text. The χ^2 99% ($\pm 2.58\sigma$) interval for the total line strength is also presented. All of our analyses support a sharp pair total line strength of about 123.5 counts, and statistically significant lower bounds equal to or greater than 23 counts and upper bounds less than or equal to 252 counts.

We emphasize that the quantitative results of the present analysis contradict in no way the quantitative empirical results of the APEX report [1]. In particular, their 99%CL upper limit of 292 total sharp pair counts ($0.572\mu\text{b}/\text{sr}$ in their Figure 2b) near 800 keV is consistent with, and in fact less restrictive¹⁵ than any of the present upper bounds upon the total

¹⁵Neither the APEX report [1] nor M. Wolanski's thesis [15], where the upper bound analysis was first published, provide sufficient detail to determine the source of the difference between their upper bound and ours.

counts¹⁶. Moreover, the APEX upper bounds of Ref. [1] exhibit a maximum near 800 keV, like that which Table I presents for the present upper bounds, which arises from the large excess of pair counts measured in the 790 keV bin. Presumably the APEX analysis would also have produced a positive 99% CL *lower* bound, also greater than that presented here, if one had been extracted.

V. DISCUSSION OF APEX POSITIVE SHARP PAIR EVIDENCE

APEX reported its data and concluded that “....the results of the present experiment represent a real disagreement with the previous observations”. Yet the present analysis of the same data shows that the APEX data corroborates the existence of sharp pairs at a confidence level exceeding 99%. How can the same data support both conclusions?

Our answer to this question divides into two parts. In the first we compare APEX’ actual sharp and background pair counts against EPOS’ in terms of strictly empirical data-only ratios of “Qualitatively Similar” quantities. We conclude that these pair ratios indicate not only no contradiction between APEX and EPOS, but in fact evidence a remarkable comparability and consistency between the EPOS and the APEX pair databases, both in size and quality. In the second we exhibit APEX’ expectations for the sharp pair count as inconsistent with the EPOS data whence it derives, and as excessively large. These deficiencies are traced to the APEX assumption that the sharp pair production cross section is constant and has the value of $5.0\mu\text{b}/\text{sr}$.

A. Purely Empirical Comparisons of APEX/EPOS Pair Databases

Given that the measured APEX data supports a sharp pair line near 800 keV, one must ask whether its observed 123 count strength is consistent with the EPOS’ sharp pair count (97 ± 38 counts, FWHM= 40 keV) at 809 keV. We here present a purely empirical positive answer in terms of data-only ratios of “Qualitatively Similar” pair counts. These ratios indicate that for “Qualitatively Similar” pair counts the APEX and the EPOS U+Th pair data bases are quite comparable both in size and quality, and they suggest no contradictions between them.

1. *The Requirement of Qualitative Similarity*

We first digress to emphasize that it in any quantitative comparison of the APEX and EPOS pair experiments which claims to be purely empirical, the APEX pairs must be of the

¹⁶Table III shows that for the analyses of the 20-, 40- and 60-keV bins, the upper bounds on the total sharp pair line strength are 217, 220, and 227 respectively, and that the chi-squared 99% interval’s upper limit is 252. We refer to them all together as specifying the range (217 to 252) of upper bounds on the sharp pair line strength.

type accepted in the EPOS experiment. We refer to this as the requirement of “Qualitative Similarity”. Without Qualitative Similarity, the expression for a ratio of comparable measured pair counts from the two experiments involves a ratio of the underlying cross sections for the pair production processes which is not a measured quantity. But for “Qualitatively Similar” processes the underlying unknown cross sections are one and the same, and their ratio is known to be equal to one, despite the fact that they are unmeasured. Thus, a violation of the requirement of “Qualitative Similarity” introduces an unmeasured ratio of two different production cross sections. If this ratio is unknown, then it defeats the calculation of the ratio¹⁷. If an additional non-empirical assumption is made about the ratio, then it becomes ambiguous whether the comparison should be viewed as addressing the original question or the additional assumption.

For example, one cannot know empirically whether the APEX’ comparison [1] of their own (1,n) pair set (in which any number n of electrons are accepted in coincidence with one positron) with EPOS’ (1,1)-only pair set (in which pairs only from events in which one and only one electron is in coincidence with the positron are accepted) ought to provide evidence on the existence of sharp pairs or simply on the differences between sharp pair production from (1,1) events and from (1,n) events. If one *assumes*, implicitly or explicitly, that the probability of sharp pairs is the same in the (1,n) events as it is in the (1,1) events, then a result can be obtained, but its reliability is contingent upon the correctness of the assumption, and the analysis can no longer be considered as purely empirical.

We therefore consider in Table IV only APEX’ counts of pairs which are of the “EPOS-type”: the leptons must be observed in opposite arms of the experiment (“RL” pairs), and the pairs must arise only from events in which one positron and one and only one electron were emitted, (“(1,1) pairs”)¹⁸.

2. EPOS’, APEX’ Qualitatively Similar Pair Databases Are Nearly Equivalent

Table IV summarizes some quantitative characteristics of the APEX and EPOS U+Th experiments, including their total luminosities and selected characteristics of their Qualitatively Similar RL(1,1) pair sets: their pair count totals and their pair counts per 20 keV near 800 keV. Also presented in the last two columns are the observed counts, N_{SP}^{800} , of excess sharp pairs and the ratio of these to background counts per 20 keV near 800 keV.

Table IV shows that despite the APEX’ $\sim 3 \times$ larger luminosity, it collects only about the same number of coincident EPOS-type pairs as EPOS, both overall and in 20 keV intervals near 800 keV. These ratios indicate that the overall size and the general shape of the APEX background pair distribution is similar to that of EPOS. It follows that in direct comparisons

¹⁷If the pair production process were well understood, theoretical information could be supplied to fix the ratio. The comparison might then be credible but could not be considered purely empirical in the strict sense of the term.

¹⁸Wedge cut pairs are not considered in this comparison because the APEX’ wedge differed somewhat from EPOS’.

of APEX' and EPOS' measured pair data, APEX can claim no clear statistical superiority over EPOS.

Besides the counts of their background distributions, the numbers of excess sharp pairs near 800 keV and their ratios to the background pair counts near 800 keV in columns five and six are also comparable for the two experiments. Thus Table IV shows that as regards both their coincident pair background distributions and their excess sharp pair distributions¹⁹, the APEX and EPOS data bases are comparable within $\sim 30\%$; Table IV therefore not only speaks against any substantial contradiction between them, but it supports their overall mutual consistency. It also precludes any claim of statistical superiority for the APEX' pair data over EPOS', and raises the question why APEX counted so few pairs (both of the smooth background and sharp 800 keV types) compared with EPOS despite its larger luminosity, a question to be discussed further below.

TABLE IV: COMPARATIVE MEASURES OF APEX and EPOS PAIR DATA BASES

	$L^{TOT}(\mu b^{-1})$	$N_{TOT}^{LR(1,1)}$	$\Delta N_{800}^{LR(1,1)}/20keV$	$N_{SP}^{800};$	$(N_{SP}^{800})/(\Delta N_{800}^{LR(1,1)}/20keV)$
APEX:	7000 ^(a)	(40.8K) ^(c)	(1480) ^(c)	$\leq 123 \pm 36^{(d)}$	≤ 0.083
EPOS:	2196 ^(b)	50K	1280	97 ± 38	0.076
(APX/EPS):	3.2	0.8	1.2	≤ 1.3	≤ 1.09

Table IV. The APEX/EPOS total luminosity (L^{TOT}) ratio inferred from the positron yields and efficiencies is substantially ($\approx 3\times$) larger than the corresponding ratios (≈ 1) of comparable pairs counted overall and per 20 keV near 800 keV. However, for the RL(1,1) pairs of the type which EPOS accepted, the APEX pair data base is somewhat smaller overall and somewhat larger near 800 keV, but not 3X larger as its luminosity would suggest. Furthermore, the ratio of the observed APEX and EPOS *sharp* pair counts near 800 keV is quite consistent with the ratio of background pairs in the same energy range. This table offers no evidence of any contradiction between APEX' and EPOS' sharp pair results. To the contrary it presents two very similar pair distributions which quite consistently provide two very similar sharp pair signals near 800 keV. The Table does raise the question why APEX counted so few EPOS-type RL(1,1) pairs as compared with EPOS. Notes to the table entries follow: (a) APEX' measured luminosity [1]; (b) EPOS' luminosity inferred in Table V by comparison of its positron data with APEX', following the method of Cowan and Greenberg [8]; (c) To guarantee Qualitatively Similar APEX/EPOS comparisons (see text) we consider only APEX pairs of EPOS' RL(1,1) type. These RL(1,1) data are taken from Ref. [15], Figures VI.2.2, and VI.2.2(c); see also Note 18; (d) See Note No.19.

¹⁹We note that the APEX sharp pairs have been extracted in the present analysis from the published APEX data, which includes (1,n) pairs not of the EPOS type. The ratios in columns five and six of Table IV, line 3, therefore violate the requirement of “Qualitative Similarity”. Since the selection of the (1,1) subset will surely reduce these pair counts, they can nevertheless provide upper bounds upon the corresponding Qualitatively Similar ratios. Clearly this ratio should be replaced by the analogous ratio for the excess RL(1,1) pairs above the RL(1,1) pair background when that data becomes available.

B. APEX' Expectations are Inconsistent, and Excessive

But if the APEX and EPOS pair databases are essentially equivalent, how does one understand the APEX' claim that its data contradicts the EPOS data? The answer is that APEX' contradiction is in fact not with the EPOS data but with APEX' own expectations, which are in fact based not upon the EPOS data, but upon a misconstrual of that data. We therefore consider here what APEX analysis did to generate its expected pair counts, what it might have done, and what the effects were. The conclusion is that APEX, purely by an inconsistent assumption, multiplied its sharp pair expectations by an order of magnitude over those which the EPOS data can actually sustain.

To set its expectations for its own results, APEX chose to epitomize the EPOS 809 keV line by the “maximal” value of the sharp pair production cross section reported²⁰ as “on the order of $5.0\mu\text{b}/\text{sr}$ ”. But APEX chose to disregard EPOS’ evidence²¹ of a beam energy dependence of the sharp pair production, and assumed that the cross section is also

²⁰This value was presented by EPOS [5], without further specification, as a rough generic value for all of its lines. More recently the EPOS/II collaboration [21] has published (also without specification of the averaging interval) the value of $1.4\mu\text{b}/\text{sr}$, which would seem to be appropriate for the 0.07 MeV/U single run beam energy spread of the original EPOS experiment.

²¹EPOS observed sharp pairs near 800 keV only in two of its five runs at different energies. One straightforward interpretation of those results would be that the sharp pair production process is beam energy dependent, and occurs *only* in a narrowly resonant energy interval which falls into the 0.02 MeV/U, ≈ 4.8 MeV, range from 5.85 MeV/U to 5.87 MeV/U. The implications of this as an alternative assumption to the APEX' are exhibited in Table V.B.

TABLE V: EXPECTED SHARP PAIR COUNTS

 A: CONSTANT $5.0\mu\text{b}/\text{sr}$ CROSS SECTION:

$N_{SP} = L^{TOT} \langle d\sigma_{SP}/d\Omega_{HI} \rangle \Delta\Omega_{2I}^{eff} G_{SP}$					
	N_{SP}^{OBS}	$L^{TOT} \times \langle d\sigma/d\Omega \rangle$	$\times \Delta\Omega_{2I}^{eff} G_{SP} = N_{SP}^{EXP}$	N_{SP}^{EXP}/N_{OBS}	
A1.APEX:	123	7000	5.0	0.0704	{2464}
A2.EPOS:	97	2196	5.0	0.0855	{939}

 B: CROSS SECTION NON-ZERO ONLY NEAR $E_R = 5.86$ Mev/U:

$N_{SP} = \tilde{L}(E_R) \Delta\Omega_{2I}^{eff} G_{SP} \int_{\Delta E_R} \langle d\sigma(E)/d\Omega \rangle dE$					
	N_{SP}^{OBS}	$\tilde{L}(E_R) \times \int \langle d\sigma(E)/d\Omega \rangle dE \times \Delta\Omega_{2I}^{eff} G_{SP} = N_{SP}^{EXP}$	N_{SP}^{EXP}/N_{OBS}		
B1.EPOS:	97	12.5K	{0.091}	0.0855	97
B2.APEX:	123	41.2K	0.091	0.0704	{264}

C: EPOS' LUMINOSITY from POSITRONS and APEX' LUMINOSITY:

$N_{e^+} = L^{TOT} \Delta\Omega_{2I}^{eff} G_{e^+} \langle d\sigma_{e^+}/d\Omega_{HI} \rangle$					
	$N_{e^+}^{OBS}$	$L^{TOT} \times \langle d\sigma_{e^+}/d\Omega_{HI} \rangle \times \Delta\Omega_{2I}^{eff} G_{e^+} = N_{e^+}$	$N_{e^+}^{OBS}/N_{e^+}$		
C1.APEX:	246K	7000	{173}	0.2031	= 246K
C2.EPOS	250K	{2196}	173	0.6580	= 250K

D: APEX and EPOS PARAMETER VALUES for SHARP PAIRS and POSITRONS:

	$\Delta\Omega_{2I}^{eff}$	$\epsilon_X \times LT$	$\times W_X = G_X$	$\Delta\Omega_{2I}^{eff} G_X$
1)APEX(SP):	6.86	1.3%	0.8	0.987
2)EPOS(SP):	7.03	1.4%	0.9	0.965
3)APEX(e^+):	6.86	3.7%	0.8	1.00
4)EPOS(e^+):	7.03	10.4%	0.9	1.00
				0.0103
				0.0704
				0.0122
				0.0855
				0.0296
				0.2031
				0.0936
				0.6580

Table V (in part A) displays the numbers of sharp pair counts expected in the the APEX and the EPOS experiments under the APEX' constant $5.0\mu\text{b}/\text{sr}$ cross section assumption, and (in part B) under the alternative assumption that the pair production cross section is non-zero only in a narrow beam energy interval lying between 5.85 and 5.87(MeV/U). Part (V.C) presents the calculation of EPOS' luminosity, and Part (V.D) summarizes the various parameter values used in the calculations. In each line the quantity extracted is placed in brackets $\{ \}$. The Table shows that APEX' constant $5.0\mu\text{b}/\text{sr}$ assumption produces an expected pair count (2464: line A1) for APEX which is an order of magnitude larger than the alternative assumption (264: line B2), and, correspondingly for EPOS a count, (939, line B2), which is an order of magnitude larger than the pair count which EPOS actually observed (97: line B1). The cause of these large expected values is discussed in the text. In part B, the table shows that when the energy integral of the pair cross section is fixed by the EPOS sharp pair count and by the EPOS luminosity density from the EPOS/APEX positron counts of part C, the APEX expectation is reduced by $\approx 10\times$ to 264, from the 2560 which APEX expected. (See Notes 1 and 8.) This value is somewhat smaller than the upper bound (292 counts) set by the APEX' experiment, and about 2.1 times larger than the sharp pair count which APEX actually observed. Part C of the table shows how the average positron cross section is obtained (line C1) from the APEX positron data, and (line C2) how the EPOS luminosity, L^{TOT} , is inferred from that cross section and EPOS' positron data. Part D of the table collects the values of the various experimental

parameters used in these calculations: the effective two-ion solid angle, ($\Delta\Omega_{2I}^{eff}$), the detection efficiencies, (ϵ_X), for sharp pairs and for positrons, the “live time” factors, (LT), the wedge factors (See Note 22.), (W_X), and the gathering powers, G_X , defined in the table. G_X gives the fraction of the emissions of type X created at the target which are actually counted by the detector. The units are μb , sr, MeV/U and combinations of them.

constant in energy. Such a constant $5.0\mu b/\text{sr}$ cross section assumption, together with the APEX’ luminosity of $7000\mu b^{-1}$ predicts²² (in Line A1, Table V) that the APEX experiment should count ~ 2464 excess sharp pair counts²³.

However, it is easy to see that this estimation is physically inconsistent. One simply applies it to compute the EPOS sharp pair count, whence the assumption derived. Using EPOS’ luminosity²⁴ of $2196\mu b^{-1}$, this calculation requires (in Line A2, Table V) that EPOS should count 939 excess sharp pairs, whereas in fact²⁵ EPOS counted only 97. Thus APEX’ constant $5.0\mu b/\text{sr}$ assumption immediately contradicts the EPOS data which it was supposed to epitomize. To make the prediction for EPOS’ pairs consistent with the actual EPOS count, the APEX’ constant cross section²⁶ would have to be reduced from $5.0\mu b/\text{sr}$ to $0.52\mu b/\text{sr}$,

²²(using appropriate parameters as given in Table V.D to characterize the apparatus.) Note also that the sharp pair wedge factors in Table V.D are computed on the assumption that the sharp pair distribution is a gaussian in the difference energy with the measured width [5,22].

²³ APEX’ published simulation provided a gaussian best fit of 2560 near 800 keV. Our value here of 2464 is equivalent to that result for the purposes of the present discussion. See also Notes 1 and 7.

²⁴This is the value calculated in Table V.C from APEX luminosity and the EPOS and APEX positron counts.

²⁵This inconsistency was first pointed out in Refs. [14], and is also noted in Ref. [8].

²⁶The APEX’ reply [23] to Cowan and Greenberg [8] expresses the opinion that “a proper discussion...should be carried out using apparatus independent cross sections”. But cross sections are not apparatus independent when the beam energy spread is larger than the range over which the cross section is constant. In the limit when the cross section is non-zero only in a narrow beam energy interval within the beam energy spread, it is the energy integrated cross section, not the cross section itself, which becomes “apparatus independent”. The useful “cross section” in this case is an effective average cross section defined to make the energy integral of the cross section over the beam spread of any given apparatus equal to the correct physical energy-integrated cross section. This cross section is obviously apparatus dependent, since it must vary inversely with the beam energy spread. In particular, a given thin target EPOS “cross section” must then be assigned a different effective value when it is measured with APEX’ thick target beam. E.g., the energy integrated value of EPOS’ pair cross section required (See Table V.B) to give its ~ 100 counts is roughly $0.1(\mu b/\text{sr})(\text{MeV}/\text{U})$. Thus, EPOS’ [5] reported “cross section” of $5\mu b/\text{sr}$ would correspond to a $0.02 \text{ MeV}/\text{U}$ beam energy interval (such as the interval from 5.85 to $5.87 \text{ MeV}/\text{U}$, where a narrow production cross section would contribute to the two EPOS runs where sharp

a value somewhat smaller than the APEX' 99% CL upper bound of $0.572\mu\text{b}/\text{sr}$.

If, instead of assuming a constant cross section, APEX assumed what the EPOS data suggests, that the cross section is non-zero only in a narrow energy range within the $0.02(\text{MeV}/\text{U})$ beam energy interval from 5.85 to 5.87 (MeV/U), then its expectations would be altered substantially. In this case EPOS' luminosity density²⁷ near $5.86\text{ MeV}/\text{U}$, $[12.5\text{K}(\mu\text{b}^{-1})(\text{MeV}/\text{U})^{-1}]$, and its 97 sharp pair counts imply (in Line B1, Table V) an energy-integrated sharp pair cross section of $0.091(\mu\text{b}/\text{sr})(\text{MeV}/\text{U})$ as the apparatus independent characteristic of the EPOS sharp pairs. From this integrated cross section, APEX' luminosity density, $[41.2\text{K}(\mu\text{b}^{-1})(\text{MeV}/\text{U})^{-1}]$, implies (in Line B2, Table V) that 264, not 2464, sharp pairs should be seen near 800 keV.

We note that in this case the average cross section from 5.85 to 5.87 (MeV/U) is $4.6\mu\text{b}/\text{sr}$, a value “of the order of” the $5.0\mu\text{b}/\text{sr}$ “maximal” cross section mentioned by EPOS [5]. The APEX' extrapolation of the $5.0\mu\text{b}/\text{sr}$ cross section to the whole of its $0.17(\text{MeV}/\text{U})$ beam energy interval then assumes an energy integrated cross section of $0.85(\mu\text{b}/\text{sr})(\text{MeV}/\text{U})$, instead of the actual value of 0.091, and effects thereby an unjustified multiplication of the energy-integrated cross section by a factor of $(5.0/4.6)(0.17/0.02) = 9.3$. Their sharp pair expectations are multiplied also by this same factor. It would appear that this extrapolation is the cause of APEX' expectation (line A1, Table V) that its $40.8\text{K RL}(1,1)$ pairs should exhibit ~ 2500 sharp pairs near 800 keV, while EPOS' 50K pairs exhibited (line B1, Table V) only 97, and of its physically inconsistent and excessively large expectation (line A2, Table V) for the EPOS observed pair count.

Thus, by assuming a constant cross section, despite EPOS' evidence for an energy dependence, and by assigning it an average value of $5.0\mu\text{b}/\text{sr}$ appropriate only for a narrow beam-energy interval, the APEX analysis multiplies its expected pair count by an order of magnitude²⁸.

pairs were observed). The more recently stated (in Table I of Ref. [21]) value of $1.4\mu\text{b}/\text{sr}$ is an appropriate average for the EPOS' full single run beam spread of $0.07\text{ MeV}/\text{U}$. The values of 0.62 and $0.59\mu\text{b}/\text{sr}$, for the full 0.16 and 0.17 MeV/U spreads of the EPOS and APEX experiments respectively, are also appropriate characterizations of the same EPOS 809 keV sharp pair count. Under these circumstances, the entire APEX discussion, which overlooks all of these possibilities, becomes ambiguous and inconsistent.

²⁷This value is equal to 5.7% (per 0.01 MeV/U) of the total EPOS luminosity. Because EPOS' five runs overlapped more at some beam energies than at others, its luminosity density varies with beam energy, as illustrated in Fig. 4. A simplified representation of EPOS' luminosity density was given in Fig. 1a of Ref. [23], which would yield an EPOS luminosity density 9% too large at $5.86\text{ MeV}/\text{U}$.

²⁸This assumption also leads APEX to estimate [24] the EPOS luminosity to be $\approx 400\mu\text{b}^{-1}$, some 5.5 times smaller than the value of $2196\mu\text{b}^{-1}$ obtained from the independent positron data in Table V. Thus, besides inflating the expectations, APEX assumption encourages the misapprehension that APEX luminosity is some 17.5 times larger than EPOS, rather than 3.2 times larger.

C. APEX' Apparent Detection Efficiency for Background Pairs is Much Smaller than EPOS'

An analysis similar to that of Table V can be used to obtain a purely empirical estimate of the ratio of EPOS' to APEX' overall detection efficiencies for smooth background pairs. Consider the values of $N_{TOT}^{LR(1,1)}$ given in Table IV, the relationship,

$$N_{TOT}^{LR(1,1)} = L^{TOT} \Delta\Omega_{2I}^{eff}(LT) \epsilon_{BP} \overline{d\sigma_{RL(1,1)}/d\Omega_{HI}}, \quad (5)$$

and the values of L^{TOT} , $\Delta\Omega_{2I}^{eff}$, and LT given in Table V.D. Then under the assumption that the ratio of the energy and angular averages, $\overline{d\sigma_{RL(1,1)}/d\Omega_{HI}}$, of the cross section for producing RL(1,1) background pairs is ≈ 1 in the two experiments²⁹, one computes the value of the ratio,

$$\epsilon_{RL(1,1)BP}^{EPOS}/\epsilon_{RL(1,1)BP}^{APEX} \approx 3.38 \quad (6)$$

and comes to the unexpected conclusion that the apparent value of EPOS' overall efficiency for RL(1,1) smooth background pairs is more than $3\times$ larger than APEX'.

D. APEX' Sharp Pair Count Imposes Small Upper Bound Upon Its Sharp Pair Efficiency

Of the several parameters assembled in Table V.D., the sharp pair efficiencies are least directly supported by laboratory measurement. Therefore, if a measured count of APEX' sharp RL(1,1) pairs near 800 keV were available, a purely empirical estimate for the ratio of EPOS' to APEX' sharp pair detection efficiency would be of special interest. However, the APEX' sharp pair count of Table IV includes pairs from (1,n) events of all n, and can therefore provide only an upper bound upon the RL(1,1) sharp pair count: $N_{RL(1,1)SP}^{APEX} \leq N_{RL(1,n)SP}^{APEX} = 123 \pm 46$. Then the calculation provides the following bound upon the apparent ratio,

$$\epsilon_{RL(1,1)SP}^{EPOS}/\epsilon_{RL(1,1)SP}^{APEX} \geq 2.2. \quad (7)$$

It is interesting that this result is in direct contradiction with the ratio $1.4/1.3 = 1.07$ of the published APEX [1] and EPOS [8] sharp pair efficiencies (tabulated in lines D1 and D2 of Table V.D). It suggests that the inconsistency between APEX' observed pair count of 123, and the value of 264 expected by the calculation of Table V may arise from the use of an erroneous value of a sharp pair detection efficiency, $\epsilon_{RL(1,1)SP}$, in the calculation of the expected value.

²⁹At this point an attempt to compare an APEX (1,n) pair count with a Qualitatively Dissimilar EPOS (1,1) pair count would fail because the ratio of the corresponding cross sections is unknown, and certainly not ≈ 1 . In the present case the cross sections are identical and one can reasonably assume that the the two averages differ from one another only slightly as a result of specific detailed differences between the two generally similar experiments.

E. Alternative: Front Layer Production of (1,1) Pairs

But an alternative possibility is that the ratios in equations (6) and (7), although truly an evidence of inconsistency between the EPOS and APEX experiments under the interpretation of equation (5), do not speak so directly to discrepant detection efficiencies, but instead may be indicating that the assumptions underlying equation (5) are inappropriate for the RL(1,1) (both background and sharp) pairs.

Specifically, evidence has been published [25] that the most probable ionization charge of a deflected³⁰ high-Z projectile is substantially larger in the layer of the target near its front face than the mean equilibrium value which obtains after the projectile penetrates deeper into the target³¹. In this context one could consider the hypothesis³² that the (1,1) pair events are sensitive to the projectile's ionization charge, and more likely to occur in the front ~ 0.02 MeV/U of target. In this case effective luminosities would have to be used in equation (5), which are reduced by factors of $\sim 0.02/0.17$, and $\sim 0.02/0.07$ for APEX' 0.17 MeV/U, and EPOS' 0.07 MeV/U thick targets, respectively. Then the ratios of apparent pair detection efficiencies in equations (6) and (7) would both be reduced by a factor, $0.17/0.07 \sim 2.4$, to the more tolerable values of 1.4 and ≥ 0.9 , respectively.

For this reason we have referred above to the efficiencies in equations (6) and (7) as "apparent" efficiencies. An additional preference by the (1,1) pair creation process for a particular narrowly resonant beam energy interval, if it occurs in combination with such a "Front Layer" preference, would require even further adjustments to the proper effective luminosities to be used when equation (5) is describing (1,1) pair events.

VI. A CURRENT PERSPECTIVE ON THE SHARP LEPTON PROBLEM

A. Substantial Experimental History

1. Sharp Positrons

Regarding the larger inferences which ought now to be drawn, we emphasize that a great variety of published data exists, not just on coincident sharp pairs, but on their forerunners, the sharp positrons from high-Z collisions. In 1983, the first report [27] of a sharp (FWHM ~ 80 keV) positron line was published by the EPOS collaboration. In 1985, evidence from six $Z_U = Z_1 + Z_2$ combinations showed persuasively [28] that the energies of these positrons were nearly independent of Z_U , and could therefore not be the sought-after positrons expected [29] from the spontaneous pair decay of the vacuum at $Z_U > Z_{crit} \approx 172$

³⁰In Ref. [25], Steibing et al. studied Pb+Au, at deflections of 35° in the lab with target thicknesses from $20\mu g/cm^2$ to $870\mu g/cm^2$.

³¹The author is grateful to Dr. T.E.Cowan for pointing out these possibilities and the results of Steibing et al.

³²More discussion is offered in Ref. [26].

(whose energies would increase rapidly with Z_U). By 1987, twenty three such positron lines had been measured and tabulated [30]. Their energies separated into three main groups, with mean energies of 255keV, 337KeV and 396KeV.

2. Sharp Pairs

By 1986, the EPOS' solenoidal spectrometer had become a pair spectrometer and reported [2] a narrow coincident (e^+e^-) pair line with summed energies near 760 keV, of which the corresponding positron projection resembled a narrow positron singles distribution previously measured with the EPOS apparatus. In 1988, the Orange group, using its new double-orange apparatus, reported [16] evidence (4σ) for a narrow (e^+e^-) line at a summed energy 815 keV and with opening angles $\theta_{+-} \approx 180^\circ$. The positron portion of this spectrum also resembled a positron line they had measured previously. Ten additional narrow (e^+e^-) pair lines were reported by Orange [17] in 1989, five others by EPOS [5] in 1990, and another seven lines by ORANGE [18] in 1993. The six EPOS lines group into three sets, near 625, 750 and 810 keV. The seventeen Orange lines group similarly, but the energies are more scattered and additional groups occur near 560 keV (4 lines) and 895 keV (2 lines).

3. Sharp Leptons from Positron Bombardments

Furthermore, Sakai [31–33], has repeatedly reported *very very* sharp (FWHM ≤ 3 keV) 330 keV electrons from thin target [$\beta^+ + U$ (or Th)] experiments following the thick target experiments of Erb, et al [34], and Bargholz, et al [35], which first evidenced such pairs³³.

B. Special Interest in Data from Alternatives Non-Heavy Ion Processes

1. Few MeV e^+/e^- on High-Z Atoms

Studies like Sakai's warrant particular interest, since they do not require a high-Z heavy ion collision and could be carried out independently in many low energy nuclear and atomic laboratories at comparatively modest expense. We have therefore analysed Sakai's data elsewhere [13,14,26] to infer a cross section $\sim 10^2$ mb to produce the composite particle source of its sharp electrons from a beam few MeV positrons on U (or Th). Furthermore, the brehmsstrahlung part of this process should be independent of the charge sign of the impinging particle, so that few MeV electrons could be as well used as positrons to check Sakai's β^+ results.

³³Other experiments [36,37] did not find sharp pairs, but Sakai's are the only thin target experiments, and the only experiments which were repeated. Also Sakai argues [38] that because of differences in the experiments these null signals do not constitute a contradiction of his results.

This situation strongly recommends experiments with beams of positrons and (equally well) of electrons in the energy range from 2 to 4 Mev, which can reliably measure a cross section of 10^2 mb to yield separately sharp ($\Gamma \leq 2.1$ keV) 330.1 keV electrons and/or (their expected partner) positrons. Such a project could verify Sakai's sharp line, and guarantee that it arises, at least in part, from the brehmsstrahlung creation process³⁴.

In addition, if Sakai's lines are ultimately to be encompassed [12] by the Quadronium Composite Particle Scenario [10,22,39] for the Sharp Lepton Problem, then the same scenario suggests [13,14,26] that photons resonantly absorbed on Uranium might produce Q_o decay pairs analogous to those which provide Sakai' sharp electrons, as we now discuss.

2. Pair Branching from Delbrück Uranium Resonances?

In particular, measurements of the *branching ratios* (specifically, to pairs vs. gamma re-emission) for the sharp resonances recently reported by Zilges, et al [40] in $U(\gamma\gamma)$ Delbrück scattering in the neighborhood of 1.8 MeV, promise to provide pivotal information on the structure of these resonant states. The question is whether all of the resonance states are conventional nuclear excited states, or whether one or more of them is a bound supercomposite $\{U, Q_o\}$ molecule in which the Q_o composite particle source of the EPOS pairs is bound to the U atom, as hypothesized [10,22] in the “Composite Particle (Q_o ?) Scenario” for the EPOS data^{35,36}.

Experimentally, these alternative states should be signatured qualitatively by their decay patterns: Q_o ought to decay preferentially to pairs, whereas nuclear dipole states decay primarily by photon emission. Therefore, if one or more of these resonances is measured to have a larger than expected pair emission branch [14,26], then further investigation would be indicated to determine whether the resonant state might be a supercomposite $\{U, Q_o\}$ state.

A parallel effort could study other U isotopes to determine whether (as the Q_o scenario suggests for the $\{U, Q_o\}$ molecule), any observed resonance is found also in the other isotopes of the same element: ^{236}U is *not* expected to exhibit the same *nuclear* excitations as ^{238}U , but it may support the same $\{U, Q_o\}$ *supercomposite molecular bound state*.

³⁴A supplementary experiment, with positrons (only) of kinetic energy in the range, $660 \leq K^+ \leq 795$ keV, could measure whether the composite particle pair source can also be created by the Recoilless Resonant Positron Absorption process first proposed in Ref [12] . Here, however, Sakai's implied cross sections [13] vary more widely and so offer less reliable estimations of the magnitude of the cross section.

³⁵This hypothesis allows the decays from such molecular bound states to exhibit positive lepton difference energies such as the EPOS U+Ta data require, and provided the first positive suggestion that the decaying particle must be charge composite.

³⁶The experimental exclusion of all possible lifetimes for such a composite particle asserted in Ref [41] emerged under analysis [11,42] to be not an experimental result at all, but a direct consequence of the unsupported assumption of one channel decay (to (e^+e^-) *only*) made in the analysis.

Experimental studies of pairs from these processes therefore promise to corroborate and extend (or to contradict and delimit) the presently very limited non-heavy-ion data evidencing sharp e^+e^- pairs. Whether in the end such experiments support or negate Sakai's sharp lines, they will do so from an independent experimental platform and they will be subject to independent verification in many laboratories. In that respect they offer an invaluable tool for escaping the impasse which seems to be developing among the heavy ion data.

C. Quadronium Scenario, and Quantum Electrodynamics

Our discussion has been guided by the Composite Particle Scenario, because it is the only hypothesis which seems able to encompass the whole range of data of the Sharp Lepton Problem. In that context, since no experimental evidence presently speaks to the specific structure of the composite particle, Occam's razor recommends the simplest assumption which does not introduce any new entity nor contradict any known fact. Since exotic (e^+e^-) quasi-bound states are unable to provide several excited states spaced at only a few hundred kilovolts, as the EPOS' line separations demand, Occam's choice falls upon the Quadronium ($e^+e^+e^-e^-$) atom. Although its binding mechanism is inexplicable within our present knowledge, it is also not possible at present to prove that it has no tightly bound states. For these reasons, we have utilized the Q_o atom, and the molecular states it might form with nuclei, as a conceptual structure to organize the the data of the Sharp Lepton Problem, and its discussion.

One day, perhaps, some data will be measured which could rule it out. Then hopefully those data will point us towards a new direction. Or perhaps one day we will begin to believe that the data really speaks in favor of the existence of Q_o . Then we can turn to the difficult, and perhaps profoundly fundamental, problem of describing it within the theory of Quantum Electrodynamics.

In the interim, we can ask whether the very existence of Q_o might already imply some contradiction, not of nuclear data, but of the data of QED. As a first step, we have inquired [13] what implications follow from the alteration of the four lepton ($e^+e^+e^-e^-$) spectrum in QED by the requirement that it exhibits one or more strongly bound state poles. The answer is that Q_o 's strongest effect upon contemporary QED would lie in its modification of the decay rate of triplet orthopositronium³⁷, a quantity already remarkable for the persistent 10σ difference [45,46] between its measured value and the best available calculation.

D. Sharp Annihilative Positron Emission

One other still untested experimental implication of the Bound Annihilative Pair Decay in the Composite Particle Scenario is the decay by Sharp Annihilative Positron Emission

³⁷ And not in the value of $(g_e - 2)$, upon which the X_o particle [43,44], which was assumed to couple directly to (e^+e^-) , was shown to have an intolerably large effect, but which Q_o influences only in order α^4 , smaller than the current experimental uncertainty [13].

[10,47] in which³⁸, in which the electron of a decay pair from a bound $\{Z, Q_o\}$ molecule is captured into an unoccupied Bohr orbit in the heavy nucleus, Z . Then only the positron emerges, carrying the total decay kinetic energy plus the added binding energy difference between the molecule and the bound electron. A bound 805 keV pair decay line would emit a series of such annihilative positrons with sharp (in the nucleus' rest frame) kinetic energies³⁹ of about $805+132=937$, 838, 820,...keV⁴⁰. The occurrence of such positrons would provide strong evidence that the source of the sharp pairs is a composite particle which can bind to a high- Z nucleus.

VII. SUMMARY AND CONCLUSIONS

The published APEX data exhibit a sharp pair line (123 ± 46 counts) of width 23.4 keV at a sum energy of 793 ± 7 keV, according to a chi-squared analysis. Confidence Level analysis of the APEX data also implies at the 99% confidence level a mean sharp pair value greater than 23 and fewer than 146 sharp pairs in the 20 keV bin at 790 keV. For our best fitting line shape, these bounds correspond to total sharp pair counts greater than 24 and less than 217. These results, and others based upon the combining of two and three bins, all agree that some sharp pair count greater than 23 and less than 252 should be expected in any repetition of the APEX experiment. All of these implications of the APEX data also honor the sharp pair upper bound (292 counts near 800 keV) inferred by APEX from the same data.

Regarding the question of conflict between the APEX and EPOS experiments, we have presented quantitative purely empirical indices of the two data bases which show that, although APEX' measured luminosity (as inferred from the total positron counts and efficiencies of the two experiments) is $\approx 3\times$ larger than EPOS', nevertheless APEX' actually counted altogether 20% fewer (40.8K/50K) "Qualitatively Similar" EPOS-type pairs than were counted by the EPOS experiment, and some 20% more (1480/1280) pairs per 20 keV near 800 keV. These ratios show that for direct APEX/EPOS comparisons of coincident pairs, the APEX data base is rather comparable to the EPOS', and certainly not substantially larger. Furthermore, the ratios of sharp pairs to background pairs near 800 keV agree within 10% for the two experiments, suggesting mutual consistency between APEX and EPOS also with regard to their *sharp* pairs near 800 keV.

Thus, under purely empirical criteria, the APEX data must be judged to be at least weakly corroborative of the EPOS' 800 keV sharp line. On the other hand, the APEX

³⁸In addition, one photon emission, in which the full ~ 1.8 MeV energy of the Q_o is emitted as a single photon, and the more elusive ("bipositron") emission of (e^+e^+) and ("tri-lepton") emission of $(e^+e^+e^-)$, can follow [12,48] Q_o decay in a bound $\{Z, Q_o\}$ state.

³⁹Ref. [10] discusses these lines in more detail and estimates their relative emission probabilities.

⁴⁰specified by the Bohr electron binding energies, where the 1s binding energy has been taken to be 132 keV.

evidence is very comparable with EPOS: no one who doubted the EPOS results needs to be convinced by those of APEX.

Consideration of the APEX collaboration’s *expectations* for sharp pairs vis a vis EPOS’ data indicates that those expectations inflate the implications of the EPOS data arbitrarily and should therefore be disregarded in assessing the experimental situation.

A puzzle emerges why, despite the fact that its luminosity is ($\approx 3.2 \times$) larger than EPOS, APEX measures roughly the same number, both of background and of sharp EPOS-type RL(1,1) pairs, as does EPOS. This means that the APEX’ Gathering Power⁴¹ for RL(1,1) pairs is not comparable to EPOS’, as one might have expected, but $\sim 2 \times$ to $3 \times$ smaller. The pair data allows us to estimate specifically that EPOS apparent detection efficiency for background RL(1,1) pairs is about $3.3 \times$ larger than APEX, and that its apparent efficiency for sharp pairs near 800 keV is at least $2.2 \times$ larger. The latter result, if taken literally, flatly contradicts the ratio of the published values, for reasons not yet understood. But this discrepancy might speak instead to the possibility that the (1,1) pair events occur more probably in the front layer of the target, where the projectile ionization charge is significantly larger than its thick target equilibrium value. In such a scenario, it is the effective values of the luminosities which would require adjustment, rather than the pair detection efficiencies.

In short, the APEX experiment provides independent corroborative evidence for a sharp (e^+e^-) pair line near 800 keV. Furthermore, the background RL(1,1) pair distributions of the APEX and EPOS experiments are roughly comparable in size and shape. In that context, the rough similarity also of their sharp pair counts can be considered as merely another aspect of an overall consistency between their distributions. Thus by purely empirical measures, APEX’ and EPOS’ pair results agree, although APEX’ RL(1,1) pair counts are $\sim 3 \times$ fewer than expected from EPOS’ and the luminosities. In the end there is no purely empirical basis in the APEX experiment to support the claim that it contradicts the EPOS’ sharp pair results.

A brief perspective review emphasizes the large body of evidence which has accumulated over the fourteen years of research into what has come to be the Sharp Lepton Problem. Non-Heavy-Ion alternative studies of few MeV positrons (or electrons) and of ~ 1.8 MeV gamma rays incident upon high-Z atoms to yield sharp pairs, and a search for Sharp Annihilative Positrons in the 600 to 900 keV range are recommended as independent and perhaps more easily repeatable avenues of investigation.

VIII. ACKNOWLEDGEMENT

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⁴¹The “Gathering Power” for a certain emission process (defined explicitly for sharp pairs in Table V) of a detecting apparatus specifies the fraction of the emissions created at the target which are actually counted by the apparatus.

IX. FIGURE CAPTIONS

Figure 1 displays the total coincident pair counts, N_{OBS} , the event-mixed background distribution, N_{BK} , and the APEX simulation calculation of the sharp pair energy distribution (multiplied by 0.1) from the decay of a source created with a cross section of $5.0\mu\text{b}/\text{sr}$. These data were first published in Figure 2a of the APEX report [1].

Figure 2 presents the data from Figure 1 in the range from 500 to 920 keV where previous experiments reported evidence for sharp pairs. The largest excess (3.2σ) above the background occurs at 790 keV. The best gaussian fit near 790 keV produces the cross-hatched addition to the background, and is described in the text. The data are tabulated in Table I.

Figure 3 presents the number of counts in excess of the background, N_{EXC} , and plots (crosshatched) the best gauss fit near 800 keV. Also plotted are the 99% CL lower (24.2) and upper (146) bounds upon the mean excess pair count in the 20 keV bin at 790 keV. For the best-fitting line shape these bounds define a total line strength between 35 and 217. The bounds for all twenty one bins are listed in Table I. Except for the 790 keV bin, all the lower bounds are negligible (≤ 2), and all of the the upper bounds are less than 100 ($\approx 3.8\sigma$).

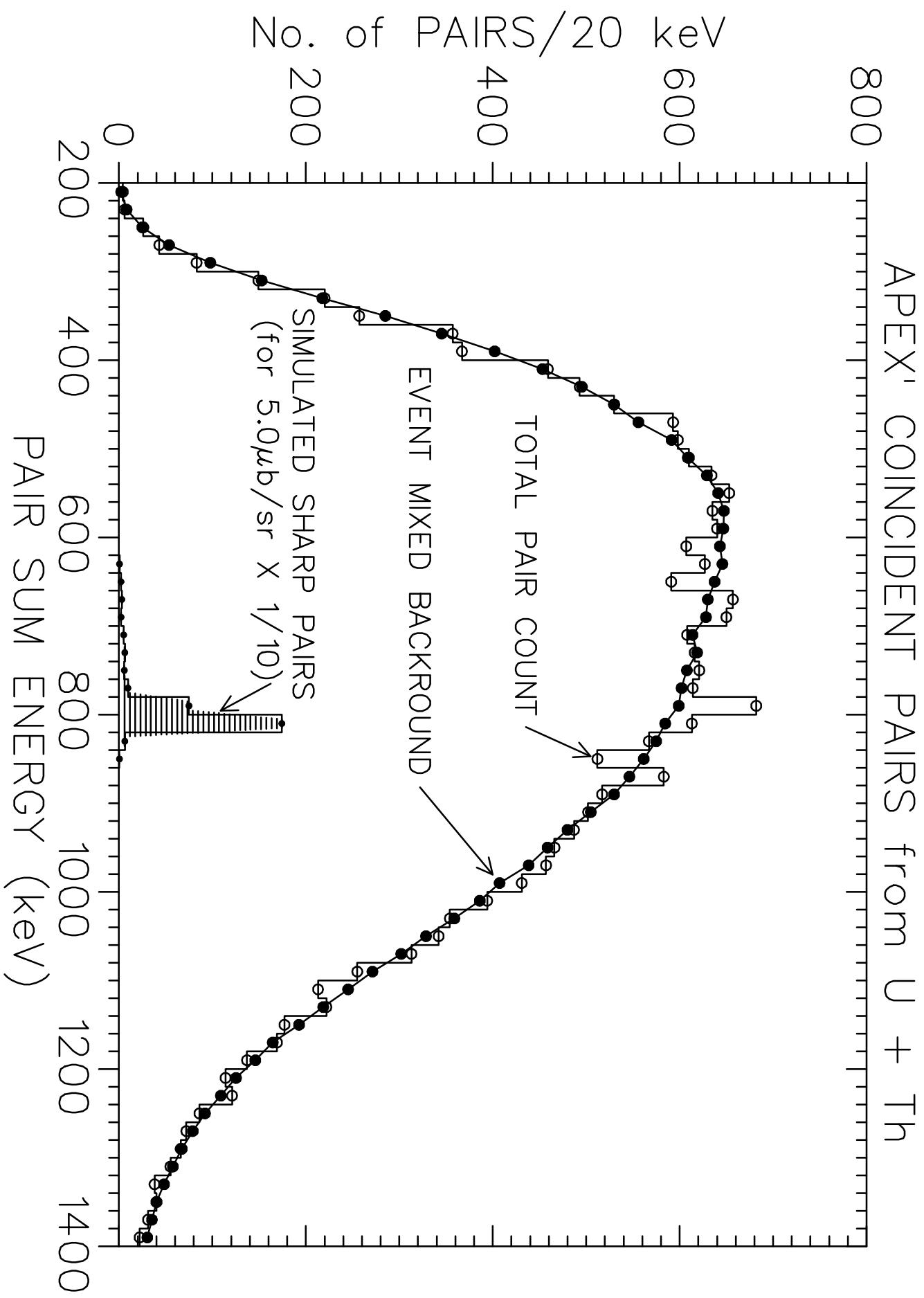
Figure 4. The normalized APEX and EPOS luminosity densities are plotted versus the beam energy. APEX' single thick target run provides a uniform luminosity density of 5.9% per 0.01 MeV/U whereas EPOS five thin target runs give the beam energy dependent luminosity shown. EPOS' 809 keV sharp pairs were seen only in their two highest energy runs, which covered the 0.07 (MeV/U) intervals up to 5.87 (Mev/U) and 5.90 (MeV/U). The fractional luminosity density in the 0.02 MeV/U interval common to these runs but not covered by any of the other three runs, where a narrowly resonant pair production cross section could have contributed pairs to the two runs in which they were observed but to no others, is 5.7% per 0.01 MeV/U.

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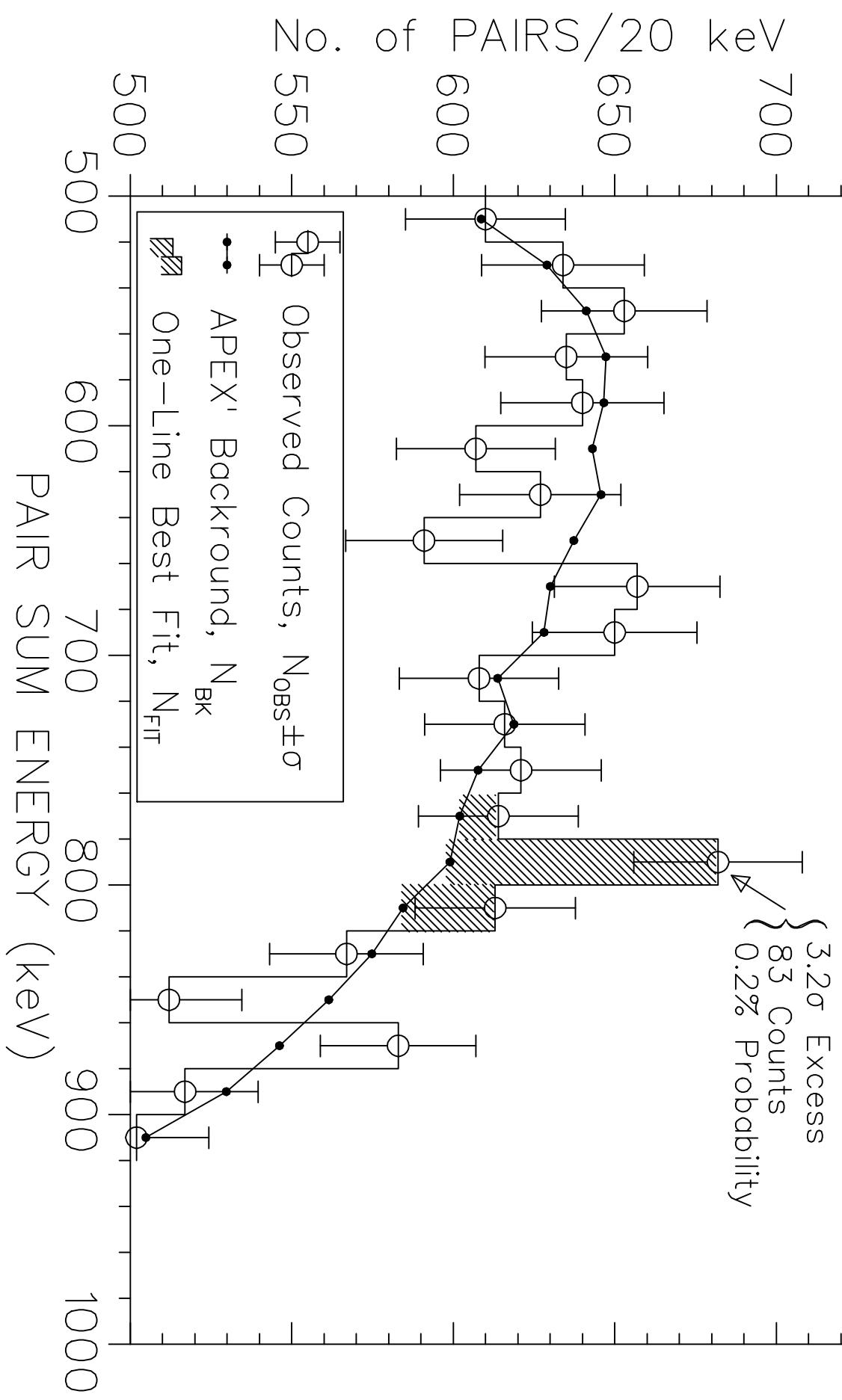
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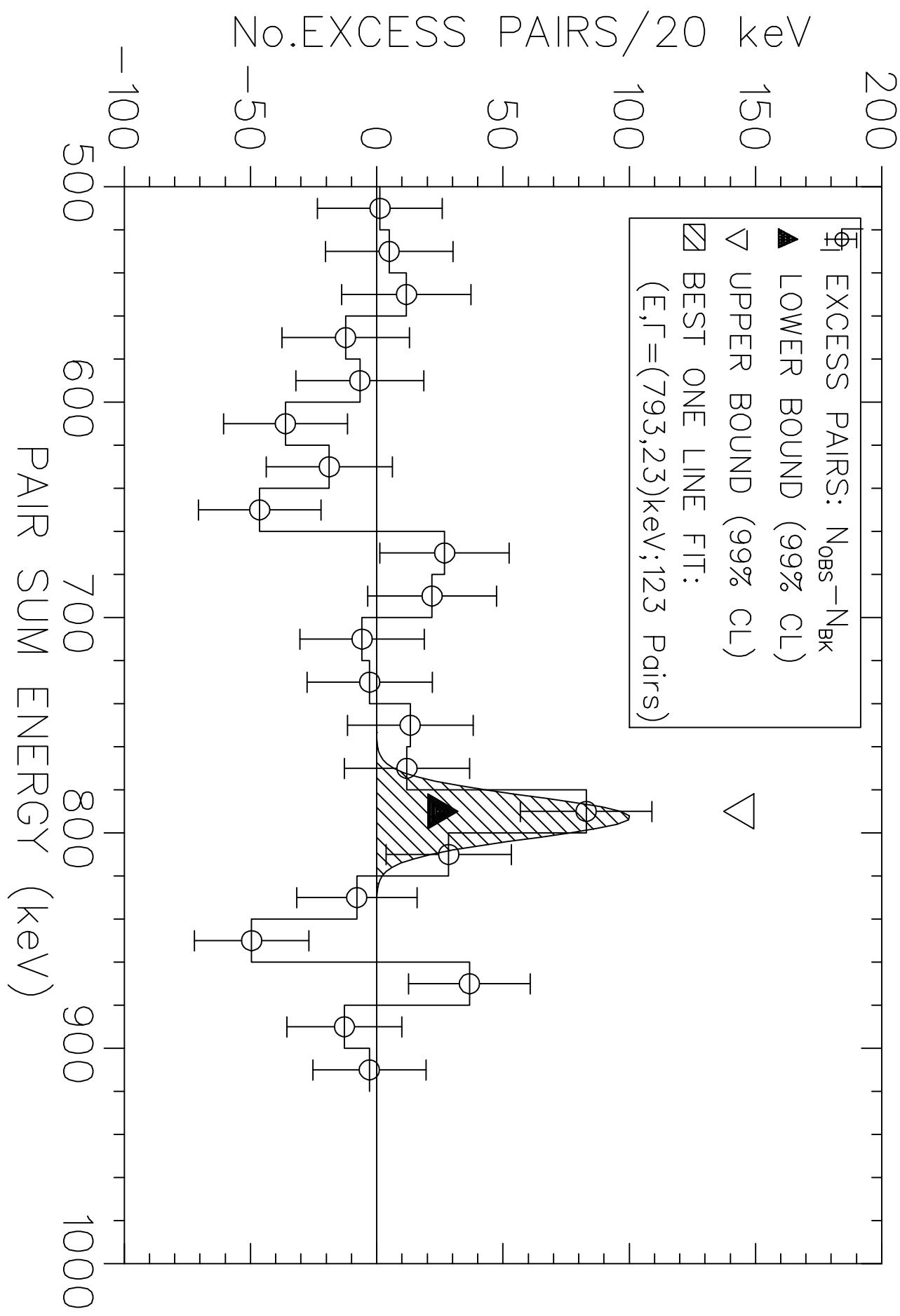
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APEX' DATA, BACKGROUND, FIT (500–900 keV)



APEX' EXCESS PAIRS: BEST FIT, 99% BOUNDS.



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